THE USE OF THE VORONOI TESSELLATION FOR PURPOSES OF SERVICE DISTRIBUTION DISTRICTS DELIMITATION (THE EXAMPLE OF THE PREŠOV NODAL REGION)

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Abstract: *The use of the Voronoi tessellation for purposes of geographical research is introduced in the paper. Spatial distribution of facilities providing services in rural areas has become a contentious topic for it reflects the hierarchy of settlements, relation between centre and its catchment area within the area. With the Prešov nodal region being the studied area the author will present the method of spatial tessellation depicting the closest located service provider.*

Key words: *the Prešov nodal region, the Voronoi tessellation, service delivery, spatial distribution.*

INTRODUCTION

The notion of space can be considered not only from geographical but also from mathematical, economical, sociological, legal and other points of view. Space as a mathematical unit is being analysed predominantly in geometry, providing rich source of knowledge exploitable in everyday practice. In the paper we try, in a simple form, to use the geometrical method of spatial tessellation to delimit the service distribution districts which can be used in further research, f. ex. delimiting catchment areas for purposes of local government reform.

The author considers the paper from the spatial, geographical point of view.

GOAL AND METHODOLOGICAL PROCEDURE

The goal of the paper is to introduce the use the method of the Voronoi tessellation in the field of geography, particularly in issues of service distribution districts. Using this method on the studied area it will be possible to analyse the districts of closest service provider.

The word „tessellation“ is derived from verb „to tessellate“ which means to cover the plane with the pieces without any gaps and covers. Words synonymously used with tessellation are tiling, paving, parqueting or mosaic. We can distinguish between homogenous or heterogenous tessellation. The figure 1 (the Escher tessellation) is an example of homogenous tessellation. The figure 2 (the Voronoi tessellation) is an example of heterogenous tessellation.

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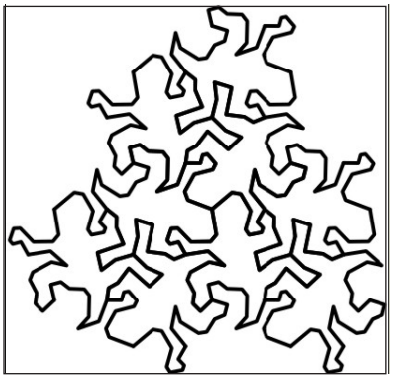
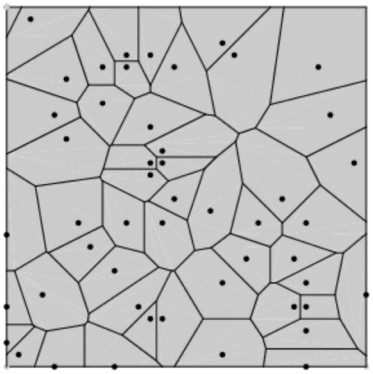
Fig. 1: *The Escher tessellation (the example of homogenous tessellation)*



Source (Ilucová 2005)

Fig.2: *The Voronoi tessellation (the example of heterogenous tessellation)*



Source (Ilucová 2005)

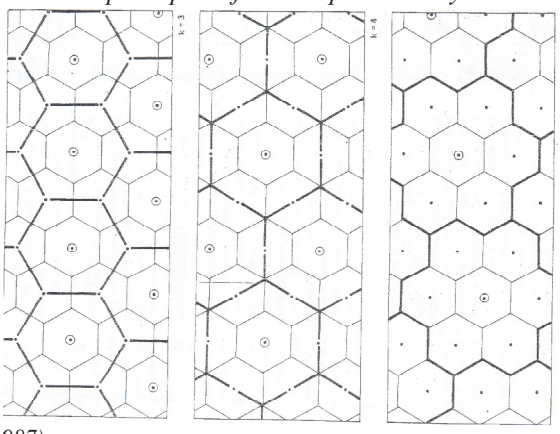
The chief principle of the Voronoi tessellation is an area (space) being divided into districts (cells). The final shape of an area consists of a set of centers to which particular parts of an area (space) are affiliated. So, the area is fragmented into a system of districts – cells entirely filling the space and having joint borders. The inside of a cell includes the points which are closer to cell´s centre than to any other centre in a plane. The borders of the cells are therefore made of points of equal distance from more centres.

The programme we work with considers the coordinates of *n* centres in a plane input information parameters. To each point in the plane, the programme affiliates the closest centre. The points affiliating to one centre constitutes the cell.

P. Ponížil (1998) claims the idea of the space division was firstly mentioned in 1644 in Descart´s work *„Le Monde de Mr Descartes, ou Le Traité de la Lumiere"*, later further explored by J. Dirichlet (1850) and G. Voronoi (1908). The author explains the ideas of tessellation were independently discovered in other scientific disciplines. In geography, the parallels of the Voronoi tessellation with central place theory can be seen. Central place theory as a theory explaining basic principles of settlement arrangement and service distribution was introduced by W. Christaller (1933). It clarifies the issues of hierarchy

of services and hierarchy of settlements. There are the two basic principles of spatial distribution of facilities – the principle of nodality (service centre – catchment area) and principle of hierarchy (various hierarchical centres). The figure 3 shows essential principles of central place theory – service principle (k - 3), transport principle (k - 4) and administrative principle (k - 7).

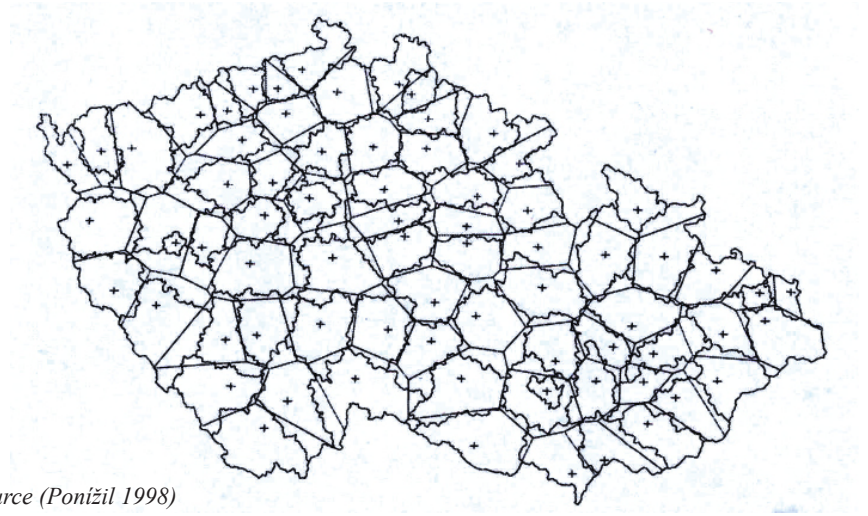
Fig. 3: *The three basic principles of central place theory*



Source (Goodall 1987)

The Voronoi tessellation can be exploitable in social sciences, f. ex. for optimalisation of administrative-territorial division (figure 4) in that it compares the current existing borders of administrative-territorial units with geometrical borders using the Voronoi method (Ponížil 1998).

Fig. 4: *Administrative-territorial units of the Czech Republic and geometrical borders using the Voronoi method*



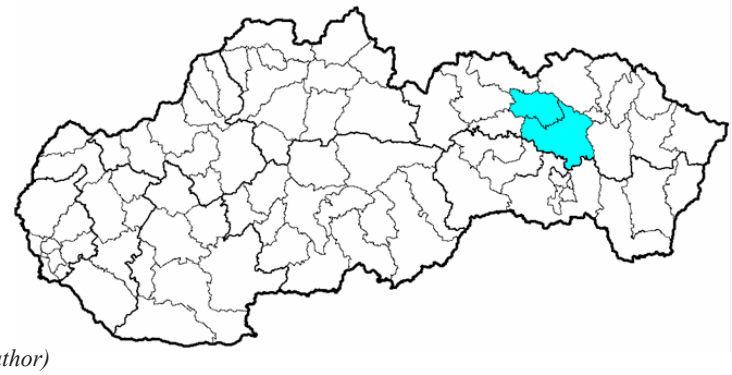
Source (Ponížil 1998)

Both the central place theory and the Voronoi tessellation operate on idealised – isotropic – environment, having common and equal features in all directions. When applying the methods to particular area the influence of relief as well as the transport communications is eliminated. It might therefore happen the programme will join communities which are not connected by any transport communication.

The spatial structure of an area limits the geographical accessibility of public and commercial services. There is a general assumption that public services must be made equally available to all (Marcuse 1978, Merget 1979 in McLafferty 1982). However, in rural areas service providers are distributed unevenly and are concentrated to certain communities. There are services with legally set districts (f. ex. postal districts, registry districts, fire and police districts) and services with no legally set districts (school, health districts). So we will point out how really existing districts vary from those constructed by the Voronoi tessellation and to what extent they are equal.

The studied area is the Prešov functional nodal region consisting of two districts – district of Prešov and district of Sabinov (figure 5). The term of functional nodal region is a purely geographical term that was introduced to Slovak geography by A. Bezák (1990, 2001). It is delimited predominantly by commute to work and school. Altogether, there are 134 communities in the studied area, four of which are considered cities – the cities of Prešov, Sabinov, Lipany and Veľký Šariš. The city of Prešov is the third largest city in Slovakia, the others are small cities having less than 10 000 inhabitants. As to communities, their size vary but, from the european point of view, they are generally small. The smallest one is Ondrášovce with 64 inhabitants, the largest is Jarovnice with more than 4500 inhabitants.

Fig. 5: *Location of the Prešov functional nodal region within Slovakia*



Source (author)

The principal asset of the Voronoi tessellation to geography is in depicting the relation of „community – closest service provider", informing about the accessibility and nodal region for the service provider. In some literature references, the districts are called "catchment areas" (Swianiewicz 2002). The procedure of constructing the maps of closest service provider is following:

1. the choice of criterion (f. ex. pharmacy),
2. the creation of database concerning coordinates of each community of studied area

being the first layer, and the coordinates of communities where pharmacies are located being the second layer,

3. the application software counts the closest distances and creates a mosaic (set of districts),
4. the result is the mosaic (set of districts) of closest pharmacy for each and every community of the studied area.

DISTRICTS OF PRIMARY SCHOOLS USING THE VORONOI ESSELLATION

The service of schooling is one of the most important toll-free public services in Slovakia. Every citizen is free to choose which primary school to attend. After the competencies have been devolved from the state bodies to local government bodies since 2002, the communities as local administrative units were given a competency of primary school securement. Primary schools in rural area are exposed to many problems regarding the demographical situation. Therefore, it cannot be surprising that due to lack of students some schools will have to be closed down in the near future. What interests us now is the spatial distribution of complete primary schools (1st – 9th classes) in studied area. In September 2006, there were 57 complete primary schools in 35 communities (incl. cities). In Prešov, there were 19 complete primary schools. The average number of students attending school was 639 (incl. cities). If we omit cities, the average number of students decreased to 284 students.

Fig. 6: *Primary schools districts in the Prešov nodal region using the Voronoi essellation*

Source (author)

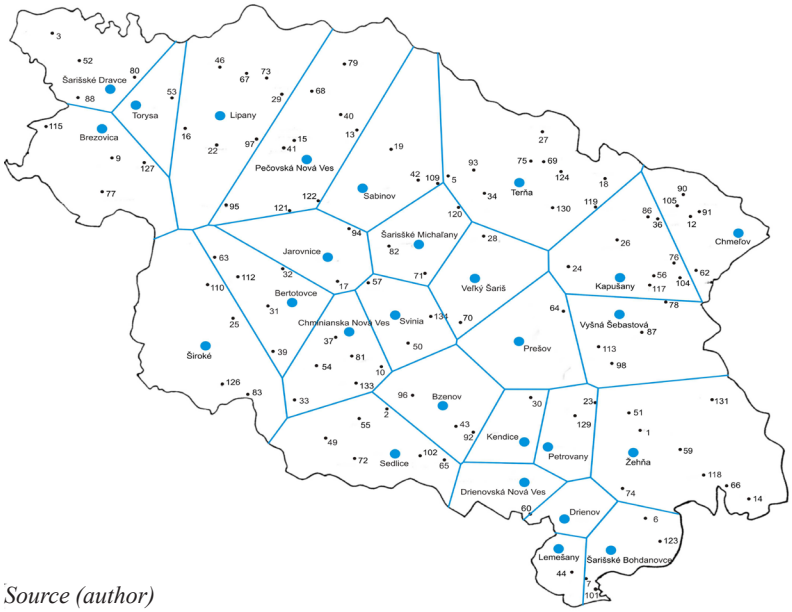
The figure 6 shows how the space (studied area) is being broken into a set of primary schools districts. We can see there are districts consisting of only one community (city of Prešov, community of Fintice and Bzenov), whereas there are districts comprising nine communities (Kapušany).

DISTRICTS OF GENERAL PRACTITIONERS (GP) USING THE VORONOI TESSELLATION

The general practitioner is free to choose in Slovakia. Recently, there has been approved a regulation of the Government of the Slovak Republic on the regionalization of the country by GPs districts. It means a person can voluntarily choose a GP, however, in case of necessity, a person belongs to legally set GP who examines them. There is also a Regulation of the Government of the Slovak Republic concerning public minimal net of health care providers in 2006 claiming their accessibility of general practitioners to 10 minutes and pediatrists to 15 minutes for 95% of population.

In the studied area, there are 72 general practitioners (46 in cities). Only 26 general practitioners are located in rural communities. The closest GP for each community is illustrated in figure 7.

Fig. 7: *General practitioners´ districts in the Prešov nodal region using the Voronoi tessellation*

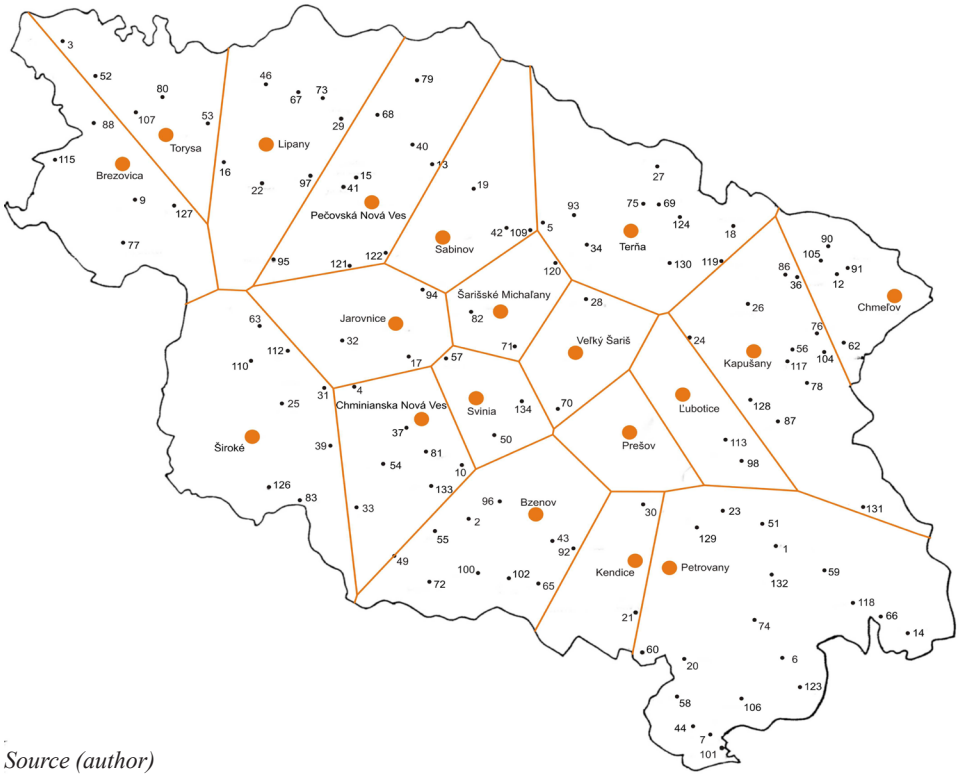


Source (author)

DISTRICTS OF DENTISTS USING THE VORONOI TESSELLATION

Similarly to GPs, people in our country are free to choose the dentist. In studied are there are 90 dentists, only 17 of which are in rural communities. There are two dentists in two communities (Kapušany and Ľubotice). The relation community - closest dentist is depicted in figure 8.

Fig. 8: *Districts of dentists in the Prešov nodal region using the Voronoi tessellation*



Source (author)

It can be seen that the districts constructed by the Voronoi tessellation in figures 6, 7 and 8 vary in size and shape as well. The size of district is understood as the number of communities to which particular service is provided. As illustrated in figure 1, central place theory considers the regular hexagon the ideal shape for delivering services. Comparing to maps most of shapes are irregular. In addition, there are unifinished districts due to spatial constraint of border as a limiting factor. Considering researched factors, some districts are totally equal - districts with the centre in Chmeľov, Šarišské Michaľany and Svinia. Other districts differentiate. Some communities can be found exactly on the border meaning they are equally far from more than one centre. It is possible to demonstrate the districts where there are strong relations among communities, however, there are communities with no transport infrastructure connection as well. The academic attention should be paid to how to make services equal for all citizens, how to prevent certain communites from being dominant in service facilities. The results are useful for many areas of social research.

CONCLUSION

Despite the method of the Voronoi tessellation has disadvantages in terms of not respecting land relief, infrastructure, borders as limiting constraint we assume it brings a lot of information for further research or implications for service-location policies. On the

example of chosen factors (primary school, GP and dentist) - the services with no legally delimited areas, we constructed ideal districts. It could also be interesting to compare the legally delimited districts with ideal districts (f. ex. postal districts, registry districts) and, if needed, to optimalise them for purposes of local administration reform.

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VYUŽITIE VORONOIOVEJ TESELÁCIE PRE ÚČELY VYMEDZENIA OBVODOV DISTRIBÚCIE SLUŽIEB (PRÍKLAD NODÁLNEHO REGIÓNU PREŠOV)

Zhrnutie

Členenie priestoru na oblasti je základným princípom Voronoiovej teselácie. Výsledkom delenia priestoru touto metódou je konečná množina centier, ktorým je priradená istá časť priestoru. Týmto je priestor rozložený na systém oblastí – buniek, úplne vyplňujúcich priestor a majúcich spoločné hranice. Vnútro buniek je potom vytvorené tými bodmi priestoru, ktoré majú k danému centru bližšie ako k iným centrám. Hranice bunky sú tvorené bodmi, ktoré sú rovnako vzdialené od viacerých centier.

P. Ponížil (1998) uvádza, že prvýkrát sa myšlienka delenia priestoru vyskytla v roku 1644 v Descartovej práci „*Le Monde de Mr Descartes, ou Le Traité de la Lumiere*“. Podrobnejšie spracovanie uskutočnili J. Dirichlet (1850) a G. Voronoi (1908) (*Voronoi diagram* alebo *Voronoi tessellation*). Autor taktiež podotýka, že Voronoiove teselácie boli objavené veľakrát nezávisle na sebe v iných vedeckých disciplínach.

Naším zámerom je poukázať na ich využitie v geografii v zjednodušenej forme. Hlavný prínos Voronoiovej teselácie pre potreby geografie verejnej správy spočíva v zobrazení väzby „obec – najbližší poskytovateľ“, ktorá podáva informáciu nielen o dostupnosti, ale i o spádovom území pre príslušného poskytovateľa. Dostupnosť tejto väzby je vyjadrená súradnicovou vzdialenosťou príslušnej obce od obce, v ktorej sídli predmetný poskytovateľ. Dôležité je upozorniť na to, že aplikačný softvér, v ktorom pracujeme, má podobne ako prostredie v TCM izotropný povrch, t.j. idealizované územie, ktoré má vo všetkých smeroch tie isté vlastnosti. Vplyv takých činiteľov ako reliéf a cestné komunikácie je eliminovaný, čo je najväčšou nevýhodou pri zavádzaní tohto postupu do reálne existujúceho priestoru. V programe, s ktorým pracujeme, sú vstupnými parametrami súradnice n centier v rovine. Program priradí každému bodu roviny najbližšie centrum zo vstupného súboru n centier. Body prislúchajúce jednému centru tvoria jednu bunku.

Postup práce:

1. Výber kritéria (napr. lekáreň).
2. Vytváranie databázy súradníc do počítača (všetky súradnice obcí ako prvá vrstva + súradnice obcí, ktoré sú sídlami lekární ako druhá vrstva).
3. Výpočet najbližších vzdialeností a tvorba obvodov v prostredí aplikačného softvéru.
4. Výsledkom je množina obvodov, ktoré zobrazujú najbližšie lokalizovanú lekáreň pre každú obec skúmaného územia.

Takto sme vytvorili tri mapy poukazujúce na najbližšieho poskytovateľa služby základného školstva, praktického lekára pre dospelých a zubára (obrázky 6, 7 a 8) každej obce v študovanom území.

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